

Advanced Algorithms Simple (for real)



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**Disclaimer**

# Course Presentation

(Usual general fluff, then come the lectures. This is the only slides-based part, found [here](https://stem.elearning.unipd.it/pluginfile.php/625081/course/section/70344/Advanced%20Algorithms%20-%20Spring%202024.pdf))

Algorithms have a general motivation: create a network of knowledge and allow, with pacing of times, different development and stories creation, while crafting new solutions. We might define them as a sequence of steps to solve the most mundane problems but also really complex problems.

There are different kinds of *applications*:

* network routing
* bioinformatics
* economics (e.g., game theory)
* fluid dynamics
* data mining
* cryptography
* machine learning

The point is this: even when making interviews, algorithms are both the logic and the solution to current problems, thinking *repeatedly and abstractly in a concrete (and fast) way*. Historically, there are still a lot of unsolved or still not found problems. That’s why the course is *mandatory*.

There are also different *goals*, wanting to introduce advanced principles of algorithm design and analysis. In particular, you’ll learn how to:

* Design algorithms for complex domains such as *graphs*
* Recognize “hard” problems and address them using *approximation algorithms*
* Use the power of *randomness* to design fast algorithms
  + and analyze them with appropriate mathematical tools

The *contents* of the course will be the following:

* (Basic) Graph algorithms
  + Graph search and its applications, minimum spanning trees, shortest paths, maximum flows 2 Approximation algorithms
* *Intractable* problems (not solvable in a reasonable amount of time)
  + NP-hardness and reductions between problems
  + Approximation algorithms for intractable problems
    - such as vertex cover, set cover, and the traveling salesperson problem
* Randomized algorithms
  + Main design techniques and analysis tools
    - with applications to problems such as sorting and minimum cuts

Although there are no formal prerequisites, an undergraduate course in algorithms and a good knowledge of (discrete) probability are assumed. Specifically, you should be familiar with:

* *Algorithm design techniques*: divide and conquer, greedy, dynamic programming
* *Data structures*: lists, stacks, queues, binary trees, search trees, heaps
* *Probability*: basic notions, discrete random variables

We want to discuss the *intuition* behind formulating algorithms and distill the core ideas making the algorithms work. Because we are computer scientists, we want to give *rigorousness*: algorithms without proofs are just conjecture and proofs give math logic to those.

We’ll follow, in part, an “active learning” approach:

* Will foster and encourage interaction during class
* Will frequently conclude class with 1-2 exercises
  + whose solution will be shown only at the beginning of next class
* Will frequently post on Moodle further readings
  + news/surveys/research articles/videos related to the topics covered in class
* There is no lab or coding assignments
  + but you are encouraged to code your favorite algorithms up and run them on real data

If you read until here, you sure wanna know: how is the exam?

* Written test, 2 hours. It consists of:
  + 3 questions
    - theory questions on the topics covered in class
    - aimed at verifying the student’s knowledge of the contents of the course
  + 2 problems
    - problems whose solution *requires some creativity*
    - aimed at verifying the student’s ability to use concepts
    - techniques learned during the course to solve new problems

# Graphs

(Suggested readings: The Algorithm, idiom of modern science [[here](https://www.cs.princeton.edu/~chazelle/pubs/algorithm.html)])

A graph is a repartition of the relationships between pairs of objects. In particular, we note:

* as the graph itself
  + = set of vertices (aka nodes)
  + (cartesian product = all) is a collection of edges
    - an edge is a pair of vertices
      * it indicates the connection between two nodes
      * a connection of vertices allows for repetition

In the following drawings, we find:

* directed graphs, which happens if
* undirected graphs, which happens if
* Immagine che contiene calligrafia, testo, inchiostro, Carattere

  Descrizione generata automaticamentearc = edge inside directed graphs (also called *directed edges*)

In this case, we’ll (mostly) use simple graphs, meaning:

* no parallel edges
* no self-loops

## Terminology and Concepts

We give some *terminology*:

* Given an edge
  + is incident on and (happens if vertex if one of endpoints in that edge)
  + and are adjacent (there is an edge between the two vertices)
* neighbors of a vertex: all vertices s.t.
  + all vertices directly connected to a given vertex by an edge
* degree of a vertex , denoted as or
  + the number of edges incident on

Examples of graphs: in many ways, graphs are the main modality of data we receive from nature.

* Road networks 🡪 (cities, roads)
* Computer networks 🡪 (computers, computers)
* World Wide Web (WWW) 🡪 (webpages, hyperlinks)
* Social networks (people, friendships relationships)
* Biological networks
  + e.g., molecules (atoms, chemical bonds)
  + e.g., brain (neurons, synapses)
* Finance 🡪 (accounts, transactions)

We give some concepts also:

* path: and
  + finite/infinite sequence of nodes which joins a sequence of vertices via edges
* simple path: (all vertices) are all distinct
  + same definition as above and vertices/nodes are all distinct/so are the edges
  + e.g., has repeated twice so it’s not simple
* cycle: simple path s.t. (starts from a given vertex/ends at same node)
* subgraph:
  + the edges of are incident only on vertices of
  + in words: it is a subset of the larger original graph
* spanning subgraph: a subgraph with
  + a subgraph which “spans” the original graph (so there are all the vertices)
  + following other definitions
    - subgraph obtained by edge deletions only but retaining all vertices
    - so it’s a subgraph of with same vertex set as
* connected graph: if a path from to
* Immagine che contiene calligrafia, Carattere

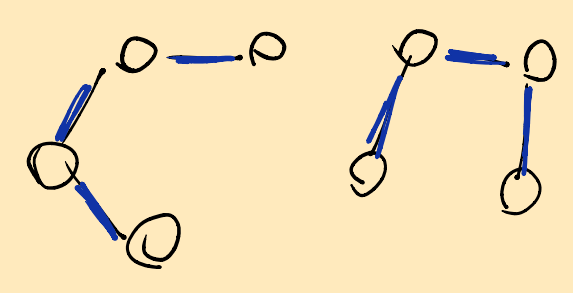
  Descrizione generata automaticamenteconnected components: a partition of in subgraphs
  + is connected
  + there is no edge between and
* tree: connected graph without cycles
  + Immagine che contiene schizzo, disegno, clipart, Line art

    Descrizione generata automaticamenteany two vertices are connected by *exactly* one path
* forest: set of trees (disjoint)
  + also = undirected graph in which any two vertices are connected by *at most* one path
* spanning tree: a spanning subgraph connected and without cycles

Immagine che contiene schizzo, disegno, Arte bambini, arte

Descrizione generata automaticamente

(it exists only if is connected)

* spanning forest: a spanning subgraph without cycles

## Basic Problems, Notations and Properties

There are different *basic problems*:

* Traversal
* Connectivity (tell if the graph is connected or not)
* Computing connected components
* Spanning trees
* Minimum-weight spanning trees
* Shortest paths

Also consider some notations and properties:

* the size of this graph is
  + is not enough (normally online you would find the size it’s = count of edges)
  + consider a scenario of a graph with vertices and no edges
  + the size of graph would be but we don’t consider vertices
    - we are accounting for both the “space” and the “connections” occupied

Exercise (Properties of graphs)

*Let be a simple, connected graph with vertices and edges. Then:*

1. *is a tree*
2. *is connected*
3. *is acyclic (i.e., is a forest)*

*Prove the previous properties*.

Solution

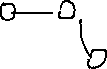
Consider an example for each one:

1. . The degree is the number of vertices incident in . The example clarifies it.



This happens because inside the summation, every edge is counted twice.

1. Consider vertices. This happens because we are choosing vertices out of to form an edge. In a simple graph, order does not matter, so we can select any two vertices, then arranging with all the possible arrangements and avoid counting each pair twice. Indeed, in a simple graph there are possible pairs of vertices.
2. If is a tree (=connected graph without cycles), consider and, you would have:



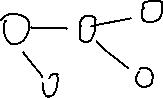
Fix a root. Then, represents father-child relationships, which are .

1. If is connected we would have



This happens because is a tree that may have cycles, thus it can only have more edges.

1. Consider an acyclic graph, then



This happens because is a tree that may not be connected, thus it can only have less edges.